Aggregate Malanobis Distance Function Approach to Evolve a HYBRID AWALE Player

Randle O. A.
Department of Computer Science, Tshwane University of Technology, Soshanguve, South Africa

Keneilwe Zuva
Department of Computer Science, University of Botswana
Gaborone

Queen Miriam Sello
Department of Computer Science, University of Botswana
Gaborone

ABSTRACT
The game of Awale is a member of the Mancala family. The problem of developing a good agent for playing Mancala games by a computer agent is an open issue. This study presents an agent that is based on the combination of Minimax search and Aggregate Malanobis Distance Function (AMDF), to evolve an agent that can play Awale at a competitive level. The result of the combination is appealing.

Keywords
Game tree, minimax search, ayo game, aggregate malanobis, Tchoukaillon strategy.

1. INTRODUCTION
The problem of developing adequate heuristics for playing mancala games by a computer is an open issue in the study of games that originate from the mancala family [1].Moreover, the development of computer game-playing algorithms is a challenging problem in artificial intelligence. Many game concepts are of practical use in problem solving and in building game-playing programs. In retrospect, the theory of games [2] looks to be explicitly designed for reasoning about multi-agent systems.

This study considers a combinatorial count-and-capture, two – person-zero-sum board game called Ayo[3].the Awale board comprises 12 pits on two rows called as usual, North and South, with 4 seeds in each pit at the beginning of a game. The following rules are commonly applied. An agent selects all seeds from a non-empty pit on his row and sows them counter-clockwise into each pit excluding the starting pit. If the last seed is sown into a pit on the opponent's row, leaving that pit with 2 or 3 seeds, the agent captures the seeds in the pit and seeds in preceding pits on the opponent's row that contain 2 or 3 seeds(this is called the 2-3 capture rule).An agent cannot capture all the seeds on the opponent's row, so he is obliged to make a move that will give his opponent a move and this is called the golden rule.A controversial rule of Awale, yet to be resolved, is when an agent cannot move in such a way that he gives his opponent a legal move, then either the game is cancelled or the agent that caused this stalemate loses the game no matter his score. The game ends(1)when an agent has captured more than 24 seeds, or (2) when both agents have captured 24 seeds leading to a draw or (3) when fewer seeds circulate endlessly on the board. Case (3) has the following specialisation: if there are fewer seeds on the board that neither agent can ever capture, but both agents will always have a legal move, the game ends and each agent is awarded the seeds on his row.

The objective of this paper is to show how minimax search can be combined with Aggregate Malanobis Distance Function(AMDF) to evolve an agent that can play Awale reasonably well. The rest of the paper is succinctly described as follows;

- Section 2 overviews the related work.
- Section 3 describes minimax search and the implementation of Aggregate Malanobis Distance Function(AMDF) procedures and provides the algorithm.
- Section 4 presents experimental test and results.
- Section 5 is the conclusion.

2. RELATED WORK
Minimax search is a fundamental class of algorithms for game playing. The algorithm constructs a game tree and uses backward induction to predict the game value. The game tree complexity is approximately W^D, where W stands for the branching factor and D is the average game length. The problem is that a combinatorial game such as Awale cannot be managed by a full game tree due to time demand and memory limitation.

An evolutionary strategy was investigated for evolving Awale, a game similar to Ayo. Six features were considered for the design of an evaluation function [4].An Awale agent utilizing minimax search was evolved using a genetic algorithm with the objective of showing that a better representation can lead to a deeper search [5].Six additional features were added to those used in [4] to improve performance of an Awale agent.

The evolved agents were evaluated against Awale shareware (Myraid Software, see website) at a search depth of 12. The results obtained at the strongest level to play are shown in Table 1[4, 5].

Table 1. Results of playing Awale using evolutionary methods

Results from Davis and Kendall(2002) at Depth 7		
Moves(Standard Deviation)	Seeds Captured(Standard Deviation)	
	Evolved Agent	Awale Shareware
80.00(5.48)	4.40(0.55)	26.80(1.64)
Results from Daoud et al.(2004) at Depth 5		

51.10(0.19)	6.40(0.27)	26.50(0.10)

The results in Table 1 provide an indication that evaluation functions with more features might not necessarily improve the performance of the Ayo agent, since the evolved agents do not considerably differ in quality when compared with Awale. An endgame database, such as constructed by [6],can be used to evolve an Awale Agent, but such an endgame database requires a large storage space. It was concluded that to improve the playing strength, it is more important to have a larger endgame database than to have a better evaluation function [7, 8].The challenge therefore, is to construct an agent with space requirements small enough to fit into main memory. Minimax search and AMDF were investigated for this purpose

3. MINIMAX SEARCH AND AMDF HEURISTIC

Generally, the value of a leaf is estimated by the evaluator and represents the number in proportion to the chance of winning the game. The evaluator can be extended to the minimax function, which determines the value for each player in a node and is formally given in (1) as follows [9,10]:

$$f(n)=\begin{cases} eval(n), & if\ n\ is\ a\ leaf\ node \\ \max\{f(c)|\, c\, is\ a\ child\ node\ of\ n\} & ,if\ n\ is\ a\max\ node \\ \min\{f(c)|\, c\, is\ a\ child\ node\ of\ n\}, & if\ n\ is\ a\min\ node \end{cases} \quad (1)$$

The function eval (n) scores the resulting board position at each leaf node n. The standard method of scoring is in terms of a linear polynomial [11]. It has been shown that every game tree algorithm constructs a superposition of a max (T^+) and a min (T^-) solution tree. The equivalent evaluator is the following Stockman equality [12]:

$$f(n)=\begin{cases} \max\left\{g(T^-) \middle|\ T^-\ is\ a\min\ tree\ rooted\ in\ n\right\} \\ \min\left\{g(T^+) \middle|\ T^+\ is\ a\max\ tree\ rooted\ in\ n\right\} \end{cases} \quad (2)$$

Where the function g is defined by [13]:

$$g(T^+)=\max\{\ f(c)|\ \ c\ is\ a\ ter\min al\ in\ T^+\ \}$$
$$g(T^-)=\min\{\ f(c)|\ c\ is\ a\ ter\min al\ in\ T^-\} \quad (3)$$

Conventionally, the basic idea of minimax algorithm is synonymously related to the following optimization procedure. Max player tries as much as possible to increase the minimum value of the game, while Min tends to decrease its maximum value at node n as both players play towards optimality. The entire process can be formally described by the following extended Stockman formula (4) below:

$$f(n)=\begin{cases} \max\{\ f(c)|\ c\ is\ a\ child\ node\ of\ n\}- f(n),\ if\ n\ is\ a\min\ node \\ \min\{\ f(c)|\ c\ is\ a\ child\ node\ of\ n\}+ f(n),\ if\ n\ is\ a\max\ node \end{cases} \quad (4)$$

The algorithm is designed in such a way that the moves (strategies) are classified into two (2) classes c_1 and c_2 which represent good and bad strategies respectively. Using the Aggregate malanobis distance function (AMDF) which finds the malanobis distance [14, 15] of each strategy on the current board state for both classes of strategies. The result of the bad strategy is then divided by the sum of both the good and bad strategies. Thereby selecting the highest possible score as the best .The AMDF algorithm is described more compactly by the following pseudo-code:

(1) Given a game state, let the vector move $[s]=\{m_1,m_2,....,m_k\}$ be a set of S feasible moves. Where there can be $s_1 \ldots s_6$ number of possible strategies

(2) Classify the moves into c_1 and c_2 classes using Tchoucallion (strategies)

(3) If c_1 and c_2 are not empty matrixes, then find the inverse (covariance) of the respective matrices which are I_g and I_b, where c_1 is the good strategies c_2 while are the bad strategies.

(4) Let $M_g = \|c_1\|$ and $M_b = \|c_2\|$ where M_g and M_b are the respective means of good and bad strategies.

(5) If $c_g = 1$ which as a determinant (mean) of c_g of good strategies which satisfies equation5below:

$$D_g = \sqrt{((s-M_g)*I_g*(s-M_g)^T)} \quad (5)$$

Else select $c_b = 0$ to satisfyequation6

$$D_b = \sqrt{((s-M_b)*I_b*(s-M_b)^T)} \quad (6)$$

(6) Compute the Distance $(D)=\dfrac{D_b}{D_b+D_g}$ where D_b and D_g are the respective distances to bad and good strategies

(7) Select the highest of the possible strategies as the best strategy using this equation for all available strategies

Furthermore we provide the mathematical explanation for our evolved player, where given data points are given as vectors $x = (x_1, x_2, x_3, ..., x_n) \in \Re^n$ and let a dataset D consists of N data points $\{x_1, x_2, x_3, ..., x_n\}$.The general problem of data clustering is to partition a dataset into m clusters of similar data points. The pd-clustering technique relates probability and distance using a simple inverse principle. For each $x \in D$ and cluster centroid c_k, the probability $p_k(x)$ that x belongs to D is given as[16].

$$\frac{p_k(x)d_k(x)}{q_k} = T \quad (7)$$

where T is a constant.

This result can be interpreted as meaning that cluster membership is more probable the closer the data point is to the cluster centroid and the larger the cluster. [26] have shown that Equation (5) is the solution of the following extremal problem:

$$\min\left\{\sum_{i=1}^{N}\left(\frac{d_1(x)p_1^{\,2}(x)}{q_1}+\frac{d_2(x)p_2^{\,2}(x)}{q_2}\right) \middle\| p_1(x)+p_2(x)=1,\ p_1(x), p_2(x)\geq 0\right\} \quad (8)$$

Where $d_1(x)$ and $d_2(x)$ are distances of the data point x to the cluster of size q_1 and q_2 and $p_1(x)$ and $p_2(x)$ are the cluster probabilities. To solve Equation (6), the Lagrangian of the problem is defined as:

$$L(P_1(x), p_2(x), \lambda) = \sum_{i=1}^{N}\left(\frac{d_1(x)p_1^{\,2}(x)}{q_1}+\frac{d_2(x)p_2^{\,2}(x)}{q_2}\right) - \lambda(p_1(x)+p_2(x)-N) \quad (9)$$

By zeroing the partial derivatives $\frac{\partial L}{\partial P_1}$ gives the solution to Equation (7) as follows:

$$P_k(x) = \frac{\prod_{j\neq k} d_j(x)/q_j}{\sum_{i=1}^{K}\prod_{j\neq i} d_j(x)/q_j} \quad (10)$$

where k is the number of clusters. The distance function $d(x, y)$ that measures the closeness of the vectors x and y is usually given as:

$$d(x, y) = \|x - y\|, \forall x, y \in \Re^n \quad (11)$$

where ||.|| is a norm. There are several norms for distance computation and examples include Chebycheu, Proscrute, Euclidean and Mahalanobis, which is preferred than the Euclidean because it is consistent across conditions and it pays equal attention to all components.

$$d(x, c_k) = \left\{(x - c_k)^T \sum\nolimits_k^{-1} (x - c_k)\right\}^{1/2} \quad (12)$$

where A^T means transpose vector of A and $\sum_K^{-1}$ is the inverse matrix of the covariance matrix $\sum_K$ given by

$$\sum\nolimits_k = \frac{\sum_i^N u_k(x_1)(x_1 - c_k)(x_i - c_k)^T}{\sum_{i=}^N u_k(x_i)} \quad (13)$$

Where

$$u_k(x_i) = \left[\left(\frac{d_1(x_i, c_1)}{q_1}\right)^2 \frac{1}{d_2(x_i, c_2)}\right] \left(\frac{d_1(x_i, c_1)}{q_1} + \frac{d_2(x_i, c_2)}{q_2}\right)^{-2} \quad (14)$$

4. EXPERIMENTAL TEST AND RESULTS

We conducted an experiment to determine the performance of our evolved player. A match consists of 10 games and each agent started 5 times. No time restrictions were given, but we accepted a default search depth of 12 for Awale so as to increase the response time. Minimax and Minimax-AMDF agents used a depth of 6.Moreover, the same computer was used by the agents, the average score and average moves were recorded.

The scores of a game are numbers of seeds captured by both agents. The agent with a higher score is the winner of a game.

Table 2. The outcome of the experiment.

ADMF-Minimax (Average)	Minimax (Average)	No of Moves
28.00	7.00	38.50
ADMF-Minimax	Awale (initiation level)	No of Moves

26.00	5	23
ADMF-Minimax	Awale (beginner level)	No of Moves
26.00	7.6	33
ADMF-Minimax	Awale (Amateur level)	No of Moves
39.6	18.6	39.6
ADMF-Minimax	Awale (Grandmaster level)	No of Moves
61.6	25	61.6

The result in Table 2 shows that Minimax-AMDF performed very well against all the opponents except at the grandmaster where it competed vigorously but lost the game at the end. The combination of Minimax and ADMF has shown that they can be combined to reasonably play Awale.

5. CONCLUSION

A heuristic is presented that combines minimax search and AMDF to evolve an Awale agent. The heuristic was tested by training an aggregate malanobis distance reasoned with strategies acquired from human agent. However, it would be interesting to investigate more extensively the AMDF training with extracts of the endgame database by[6], since the Awale game is similar to Ayo, and Awale is solved using the endgame database. The results of the experiment performed shows that combining minimax search with AMDF improves the playing strength of the Awale agent, it provides a good result at a deeper search, and give an efficient heuristic for evolving an Awale agent.

6. REFERENCES

[1] Donkers H.H.L.M., Uiterwijk J.W.H.M. and Voogt A.J.D.V. 2002 Mancala Games- Topics in Artificial Intelligence and Mathematics. Step by Step Proceedings of the 4th Colloquium Board Games in Academia.

[2] Von Neuman and Morgenstern . 1944 Theory of games and economic behaviour, Princeton, NJ.

[3] Adewoye T. O. 1990 On Certain Combinatorial Number Theoretic Aspects of the African Game of Ayo.AMSE REVIEW, vol.14, no, 2, pp.41-63,1990

[4] Davis J. E. and Kendall G. 2002 An Investigation, using co-evolution, to evolve an Awari Player. In proceedings of Congress on Evolutionary Computation (CEC 2002), pp. 1408-1413.

[5] Daoud M., Kharma N., Haidar A. and Popoola J. 2004 Ayo, the Awari Player, or How Better Representation Trumps Deeper Search, Proceedings of the 2004 IEEE Congress on Evolutionary Computation, pp. 1001-1006.

[6] Romein J.W. and Bal H. C. 2002 Awari is Solved. International Computer games Association Journal,25(3),162-165.

[7] Allis V., Muellen M. V. D. and Herik J. V. D. 1994 Proof-number Search, Artificial Intelligence, vol. 66, pp. 91-124.

[8] Lincke T. R. and Marzetta A. 2000 Large endgame databases with limited memory space. International Computer Games Association Journal, 23(3) ,131-138.

[9] Bruin A. D., Pijls W. and Plaat A. 1994 Solution Trees as a Basic for Game Tree Search, ICCA Journal, 17, 4, pp. 207-219.

[10] Bruin A. D. and Pijls W. 1996 Trends in Game Tree Search. SOFSEM, pp. 255-274.

[11] Samuel A. L. 1959 Some Studies in Machine Learning Using the Game of Checkers, IBM J. of Res. And Dev. 3, 210-229.

[12] Pijls W. and Bruin A. D. 2001 Game Tree Algorithms and Solution Trees, theor. Compt. Sci., 252 (1-20: pp. 197-215.

[13] Mahalanobis P. C. 1936 On the generalized distance in statistics. Proceedings of the national Institute of Science of India, pp. 49- 55.

[14] Maesschalck De. R., Jouan-Rimbaud D., Massart, D. L. 2000 The Malanobis distance. Chemometrics and Intelligent Laboratory Systems, pp. 1-18.

[15] Iyegun C. and Ben-Isreal A. 2008 Probabilistic Distance Clustering Adjusted for Cluster Size. Probability in the Engineering and Informational Sciences, 22, 603-621.